

November 24

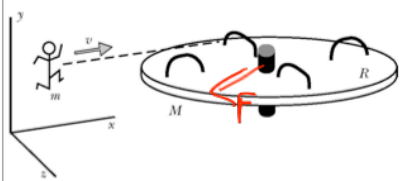
Get clickers and whiteboards

Clicker Questions: Merry-Go-Round
Q3

$$p_i = mv$$

Child runs and jumps on playground merry-go-round. For the system of the **child + disk** (excluding the axle and the Earth), which statement is true from just before to just after impact?

(K = total kinetic energy,
 \vec{P} = total linear momentum,
 \vec{L} = total angular momentum about the axle)



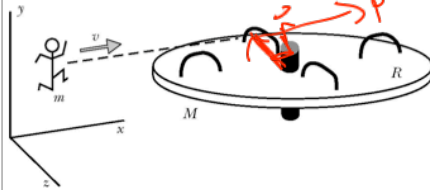
- A) K , \vec{P} , and \vec{L} do not change
- B) \vec{P} and \vec{L} do not change
- ☒ C) \vec{L} does not change
- D) K and \vec{P} do not change
- E) K and \vec{L} do not change

Inelastic $\Rightarrow \Delta K \neq 0$
 $\Delta \vec{P} \neq 0$

$\vec{L} = 0 \Rightarrow \Delta \vec{L} = 0$

Q4

What is the initial angular momentum of the child + disk about the axle?



- A) $\langle 0, 0, 0 \rangle$
- B) $\langle 0, -Rmv, 0 \rangle$
- C) $\langle 0, Rmv, 0 \rangle$
- D) $\langle 0, 0, -Rmv \rangle$
- E) $\langle 0, 0, Rmv \rangle$

$$|\vec{L}| = |\vec{r} \times \vec{p}| = |\vec{r}| |\vec{p}| \sin \theta$$

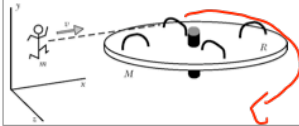
$$= Rmv \sin 90^\circ = Rmv$$

$$\vec{r} \text{ in } -\hat{z} \quad -\hat{z} \times \hat{x}$$

$$\vec{p} \text{ in } \hat{x}$$

Q5

The disk has moment of inertia I , and after the collision it is rotating with angular speed ω . The rotational angular momentum of the disk alone (not counting the child) is



A) $\langle 0, 0, 0 \rangle$

B) $\langle 0, -I\omega, 0 \rangle$

C) $\langle 0, I\omega, 0 \rangle$

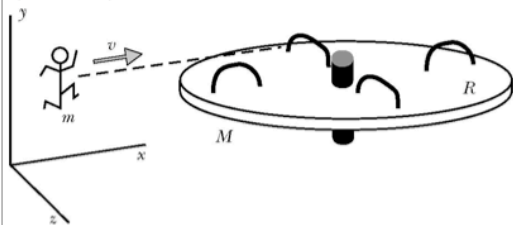
D) $\langle 0, 0, -I\omega \rangle$

E) $\langle 0, 0, I\omega \rangle$

$$\vec{L} = I\omega$$

Q6

After the collision, what is the speed (in m/s) of the child?



A) ωR

B) ω

C) ωR^2

D) ω/R

E) $\omega^2 R$

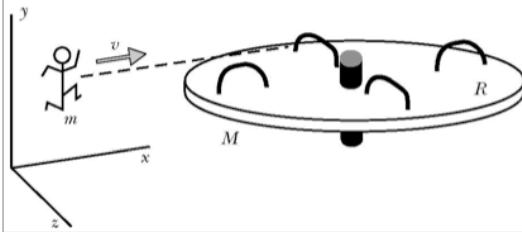
$$\omega = \frac{v}{R}$$

$$v = \omega R$$

$$R = m$$

Q7

After the collision, what is the translational angular momentum of the child about the axle?



A) $\langle 0, 0, 0 \rangle$

B) $\langle 0, -Rm\omega, 0 \rangle$

C) $\langle 0, Rm\omega, 0 \rangle$

D) $\langle 0, -Rm(\omega R), 0 \rangle$

E) $\langle 0, Rm(\omega R), 0 \rangle$

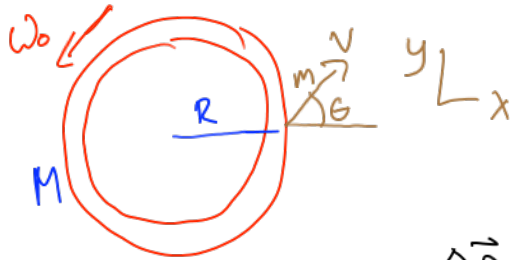
$$|\vec{r} \times \vec{p}|$$

$$\uparrow \quad \uparrow$$

$$R \quad m(\omega R)$$

Example: Space Station

A space station has the form of a hoop of radius R with mass M . Initially its center of mass is not moving, but it is spinning with angular speed ω_0 . Then a small package of mass m is thrown by a spring-loaded gun toward a nearby spacecraft; the package has a speed v after launch. Calculate the center-of-mass velocity of the space station (V_x and V_y) and its rotational speed ω after launch.

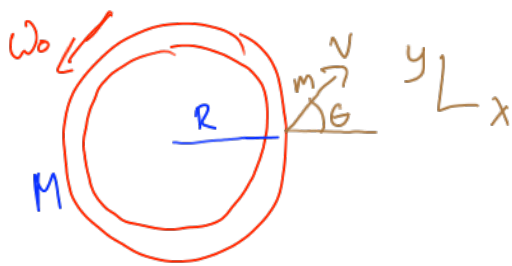


Pick as system the
space station and package

$$\vec{F}_{\text{net,ext}} = 0, \quad \vec{\tau}_{\text{net,ext}} = 0$$

$$\Delta \vec{p} = \vec{F}_{\text{net,ext}} \Delta t = 0 \Rightarrow \vec{p}_f = \vec{p}_i$$

$$\Delta \vec{L} = \vec{\tau}_{\text{net,ext}} \Delta t = 0 \Rightarrow \vec{L}_f = \vec{L}_i$$



$$\vec{P}_f = \vec{P}_i$$

$$P_i = 0$$

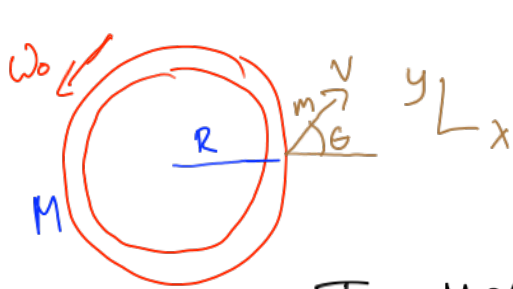
$$M\vec{V} + m\vec{v} = \langle 0, 0, 0 \rangle$$

$$\langle MV_x, MV_y, MV_z \rangle + \langle mv \cos \theta, mv \sin \theta, 0 \rangle = 0$$

$$V_x = -\frac{m}{M} v \cos \theta$$

$$V_y = -\frac{m}{M} v \sin \theta$$

$$V_z = 0$$



$$\vec{L}_f = \vec{L}_i$$

$$\vec{L}_i = I \omega_0 \hat{z}$$

$$I = MR^2 \quad \vec{L}_f = I \omega \hat{z} + R m v \sin \theta \hat{z}$$

$$I \omega + R m v \sin \theta = I \omega_0$$

$$\omega = \omega_0 - \frac{R m v \sin \theta}{I} = \omega_0 - \frac{m v \sin \theta}{M R}$$

Clicker questions: Quantization of angular momentum

Q1

In the original Bohr model of the hydrogen atom, the electron moves in circular orbits around the proton. Which of the following equations is correct for this model? (More than one choice may be correct; pick one.)

A) ~~$\left| \frac{d\vec{p}}{dt} \right| = -\frac{1}{4\pi\epsilon_0} \frac{e^2}{r}$~~

B) $\left| \frac{d\vec{p}}{dt} \right| = \frac{1}{4\pi\epsilon_0} \frac{e^2}{r^2}$

C) $\left| \frac{d\vec{p}}{dt} \right| = \frac{v}{r} mv$

D) ~~$\left| \frac{d\vec{p}}{dt} \right| = G \frac{Mm}{r^2}$~~

E) ~~$\left| \frac{d\vec{p}}{dt} \right| = -\frac{1}{4\pi\epsilon_0} \frac{e}{r^2}$~~

$$\left| \frac{d\vec{p}}{dt} \right| = |\vec{F}_{\text{net}}|$$

$$\vec{F} = \frac{1}{4\pi\epsilon_0} \frac{q_1 q_2}{r^2}$$

Q2

For the bound states of hydrogen, which statement is true?

- A) $K+U$ is positive
- B) $K+U$ is negative
- C) $K+U$ is zero

Q3: Which is the correct expression for $K+U$ for the hydrogen atom?

A) $\frac{1}{2}mv^2 + \frac{1}{4\pi\epsilon_0} \frac{e^2}{r}$	C) $\frac{1}{2}mv^2 + \frac{1}{4\pi\epsilon_0} \frac{e^2}{r^2}$
B) $\frac{1}{2}mv^2 - \frac{1}{4\pi\epsilon_0} \frac{e^2}{r}$	D) $\frac{1}{2}mv^2 - \frac{1}{4\pi\epsilon_0} \frac{e^2}{r^2}$
	E) 0

$$K = \frac{1}{2}mv^2$$

$$U = \frac{1}{4\pi\epsilon_0} \frac{q_1 q_2}{r}$$

$$e(-e)$$

Q4: Starting from the notion that the angular momentum of the electron is quantized, Bohr arrived at the following formula for the radius of the circular orbit:

$$r = \frac{\left(\frac{N^2 \hbar^2}{m} \right)}{\left(\frac{1}{4\pi\epsilon_0} e^2 \right)}$$

What does this predict for the numerical value of r ? (Leave N^2 as a factor.)

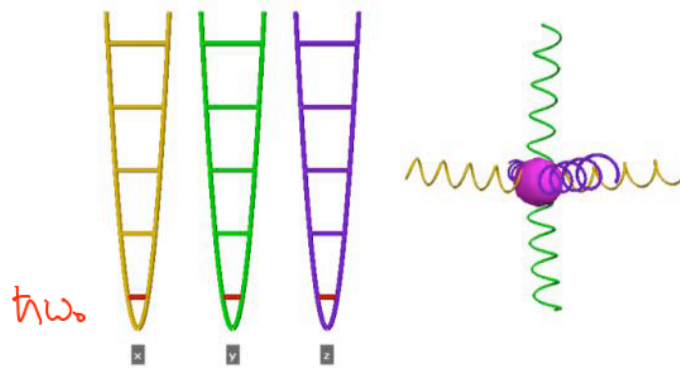
- A) $r = (8.5\text{E-}30 \text{ meter})N^2$
 B) $r = (5.0\text{E+}23 \text{ meter})N^2$
 C) $r = (4.8\text{E-}1 \text{ meter})N^2$

- D) $r = (5.3\text{E-}11 \text{ meter})N^2$
 E) $r = (1.2\text{E-}38 \text{ meter})N^2$

$\hbar = 1.05\text{E-}34 \text{ J} \cdot \text{s}$, $m = 9\text{E-}31 \text{ kg}$, $e = 1.6\text{E-}19 \text{ C}$, $\frac{1}{4\pi\epsilon_0} = 9\text{E9} \text{ N} \cdot \text{m}^2 / \text{C}^2$

Chapter 11: Quantum Statistical Mechanics

Einstein Solid



$h\omega$

unit of quant $h\omega$

$$\omega = \sqrt{\frac{k_s}{m}}$$

$K + U$

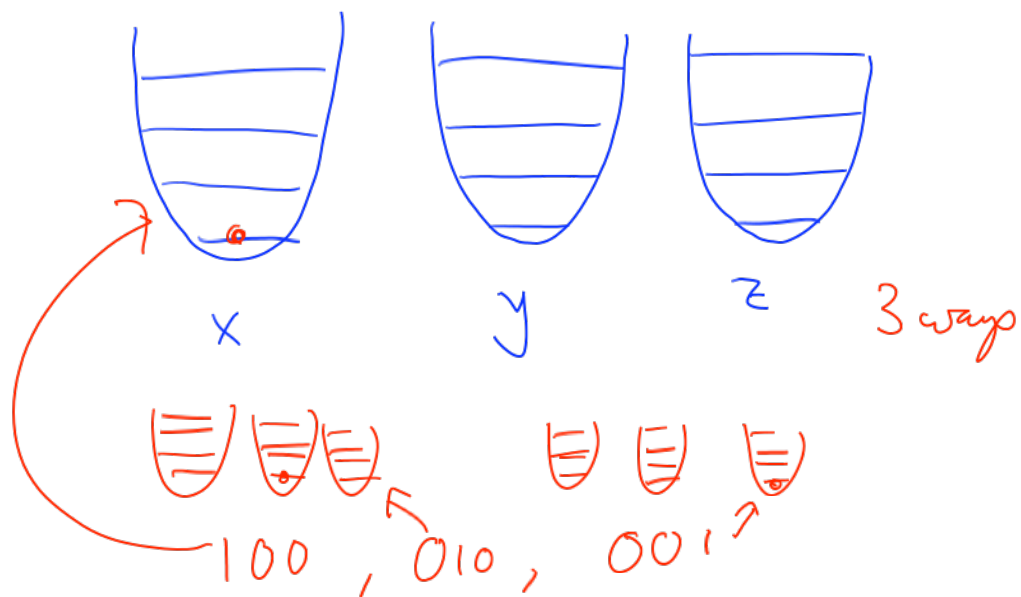
$$K = \frac{1}{2} m v^2$$

$$U = \frac{1}{2} k_s S^2$$

$$V^2 = v_x^2 + v_y^2 + v_z^2$$

$$S^2 = x^2 + y^2 + z^2$$

How many different ways can one quantum of energy go into a system of three oscillators (one atom)?



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Tangible:

$$\Omega = \frac{(q+N-1)!}{q!(N-1)!}$$

A: List the different ways 2 quanta of energy can go into a system of 3 one-dimensional oscillators

$$\frac{(2+3-1)!}{2!(3-1)!} = \frac{4 \cdot 3 \cdot 2 \cdot 1}{2 \cdot 1 \cdot 2 \cdot 1} = 6$$

200, 020, 002,
110, 101, 011

6 ways

B: List the microstates for 2 quanta of energy shared among 4 one-dimensional oscillators

$$\frac{5!}{2!3!} = \frac{5 \cdot 4 \cdot 3 \cdot 2}{2 \cdot 1 \cdot 3 \cdot 2} = 10$$

1100, 1010, 1001, 0110, 0101, 0011
2000, 0200, 0020, 0002

10 ways

C: List the microstates for 4 quanta of energy shared among 2 one-dimensional oscillators

$$\frac{5!}{4!1!} = 5$$

04, 40, 31, 13, 22

5 ways

$$\Omega = \frac{(q+N-1)!}{q!(N-1)!}$$

$$N! = n(n-1)(n-2)\dots 3 \times 2 \times 1$$

$q = \#$ of quanta

$N = \#$ of oscillators

$\Omega = \#$ of ways

$N=6, q=4:$

Fundamental Assumption of Statistical Mechanics

all possible "microstates" are equally probable

0+4

1+3

2+2

3+1

4+0

Table	4+0	3+1	2+2	1+3	0+4
	1	2	3	4	5
# of ways	15	30	36	30	15

$$\Omega_3 = \frac{(3+3-1)!}{3! 2!} = 10$$

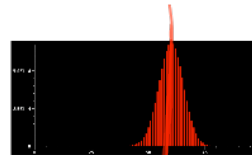
$$\Omega_1 = \frac{(1+3-1)!}{1! 2!} = 3$$

$$\Omega = 10 \times 3 = 30$$

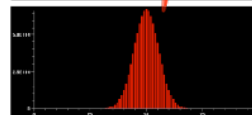
Bar chart from Excel

VPython version of bar chart

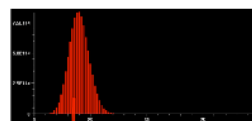
$N = 500, N1 = 300, N2 = 200, q = 100$



$N = 500, N1 = 250, N2 = 250, q = 100$



$N = 500, N1 = 100, N2 = 400, q = 100$



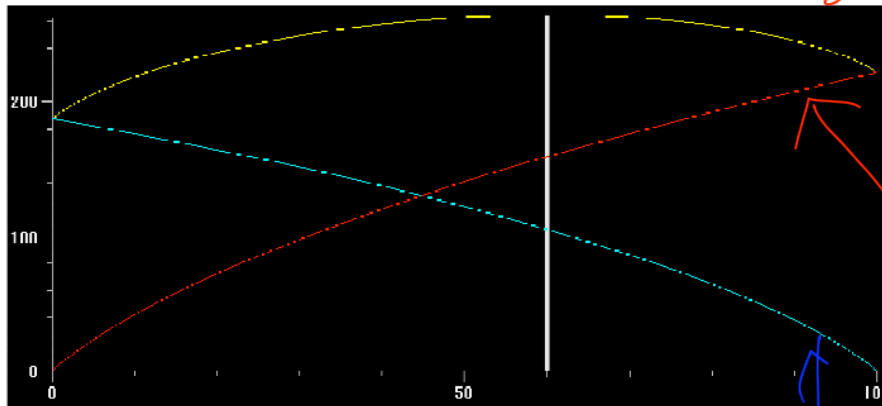
$N = 500, N1 = 400, N2 = 100, q = 100$



How do we handle huge numbers?

Take logs

$\ln(\text{total \# of ways})$



log of
in N

log(N)

$$S = k \ln(\text{\# of ways})$$

Entropy

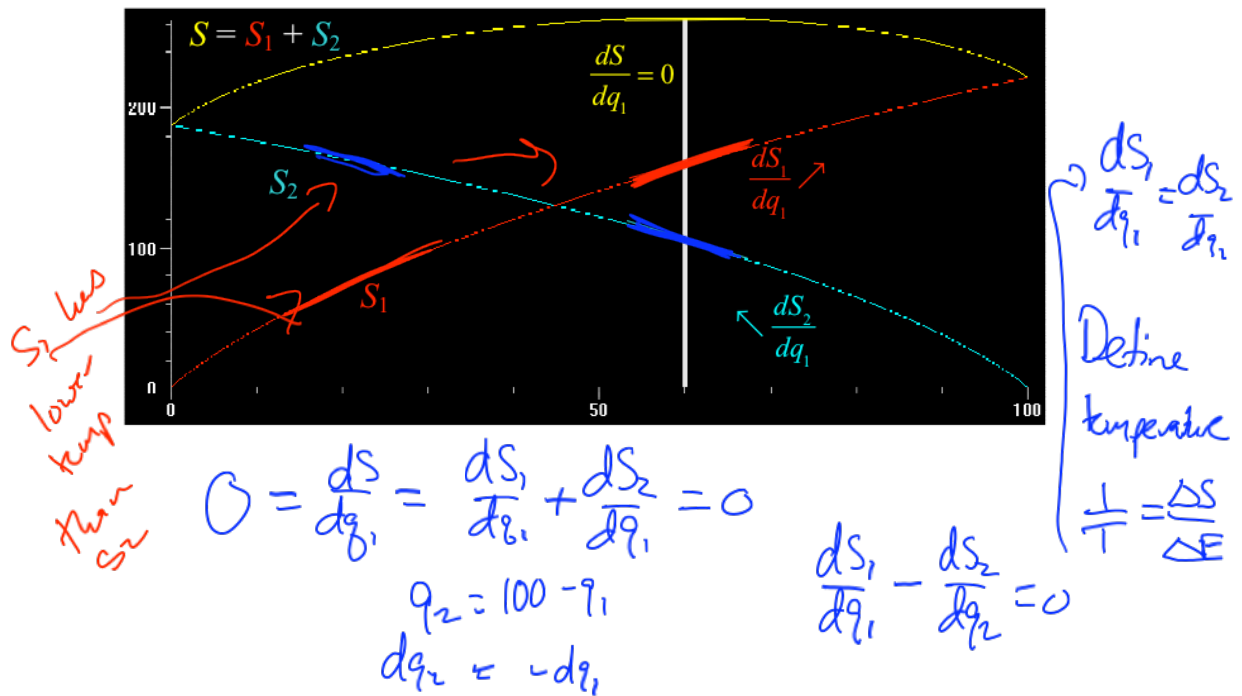
$$k = 1.38 \times 10^{-23} \text{ J/K}$$

$$\begin{aligned} \ln(\text{total}) &= \ln(N \times M) \\ &= \ln(N) + \ln(M) \end{aligned}$$

2nd law of Thermodynamics

If a ^{closed} system is not in equilibrium, the most probable consequence is that the entropy will increase.

Second Law of Thermodynamics



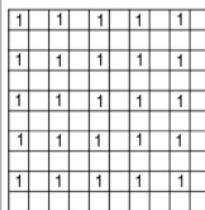
Q1

A system of 300 oscillators contains 100 quanta of energy. What is the physical meaning of this model?

- A) one atom oscillating in 300 dimensions
- B) 300 atoms, each in the 100th energy level
- C) 300 atoms with 100 joules of energy distributed among them
- D) 100 atoms with 300 joules of energy distributed among them
- E) 100 atoms with $100\hbar\sqrt{\frac{k_s}{m}}$ joules of energy among them

Q2

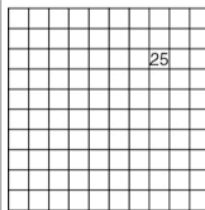
10 by 10 array of 100 oscillators, 25 quanta of energy



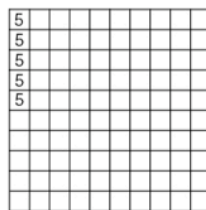
A



B



C



D

Which arrangement is most probable?

- A) A
B) B
C) C
D) D
E) They're equally probable

Q3

Inside an insulated container two aluminum blocks, 1 kg and 2 kg, have been in contact for a long time. What physical property is the same for the two blocks?

- A) the mass
- B) the temperature
- C) the volume
- D) the thermal energy
- E) the weight